Technical Comments

Comment on "Optimal Space Flight with Multiple Propulsion Systems"

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In a recent article, Hazelrigg[1] presented a thrust switching criterion that may be used to select the optimum thrust and I_{sp} levels (out of several available combinations, including coast) at all points along a space flight trajectory. A conventional "switching function" K_i is computed for each of the n possible thrust combinations; the value of $i(1 \leq i \leq n)$ is then chosen to maximize the product $K_i\beta_{i\ max}$ or, equivalently, the Hamiltonian function H.

This is an interesting and useful approach. It must be pointed out, however, that essentially similar contributions have appeared previously in the 1967–1968 literature. The object of this Comment is to bring two prior and relatively thorough treatments to the attention of readers who may be contemplating further development of the subject. Papers by Dickerson and Smith,[2] and Fishbach,[3] for example, resulted in switching criteria identical or mathematically equivalent to the $K_i\beta_i$ function of Ref. 1 [i.e., compare Fig. 1 (Fig. 1 of Ref. 1) with Fig. 2 (Fig. 4a of Ref. 3)]. Moreover, Ref. 2 analytically developed the optimal thrust sequencing policy; the same result was illustrated numerically (for several values of n) in Ref. 3. References 1 and 2 both showed a solar-electric power source in their illustrative examples, while Ref. 3 showed the constant power case. Reference 3 further compared the resultant acceleration histories, J values, and propellant requirements with those corresponding to the variable-thrust case, and showed that variable thrust performance can be rather closely approached using as few as three different thrust levels.

The main features of these contributions are summarized in Table 1. We suggest that the points covered are well known by now and that future contributions could more appropriately be directed toward either significant theoretical extensions or realistic engineering applications.

Table 1 Some recent results on multiple-thrust-level trajectories

	Paper		
Result	Hazelrigg,[1] 10/68	Dickerson and Smith,[2] 8/68	Fishbach,[3] 8/67
Development of βK criterion (or equivalent)	Yes	Yes	Yes
Thrust sequencing policy	Analytical: illustrative analytic example for the field-free case, $n = 2$	Analytical proof	Numerical demonstration for n = 2, 3, 5, 6
Power source illustrated	Solar-electric	Solar-electric	Constant power
Efficiency illustrated	Constant	Variable	Not considered
Comparison with variable thrust case	No	No	Yes; $a(t)$, $a^2(t)$, $J(t)$, and M_p as functions of n

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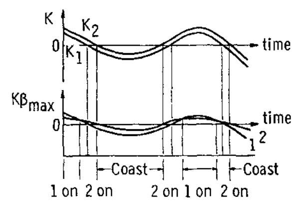

Fig. 1 Switching function histories taken from Ref. 1.

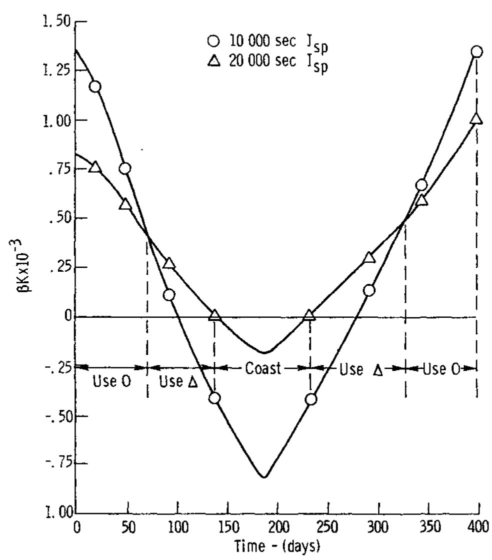

Fig. 2 Switching function histories taken from Ref. 3.

References

[1] Hazelrigg, G. A., Jr., "Optimal Space Flight with Multiple Propulsion Systems," *Journal of Spacecraft and Rockets*, Vol. 5, No. 10, Oct. 1968, pp. 1233–1235.

[2] Dickerson, W. D. and Smith, D. B., "Trajectory Optimization for Solar-Electric Powered Vehicles," *Journal of Spacecraft and Rockets*, Vol. 5, No. 8, Aug. 1968, pp. 889–895.

[3] Fishbach, L. H., "Multiple Thrust Level Trajectories for Minimum Propellant Consumption," M.S. thesis, 1967, Case Institute of Technology, Cleveland, Ohio; also TM X 52322, Aug. 1967, NASA.

Erratum: "Natural Frequency of Longitudinal Modes of Liquid Propellant Space Launch Vehicles"

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In the foregoing paper expressions were derived for use in calculating longitudinal modes of thin shell tanks that are built up from conical frustums and which contain liquid.

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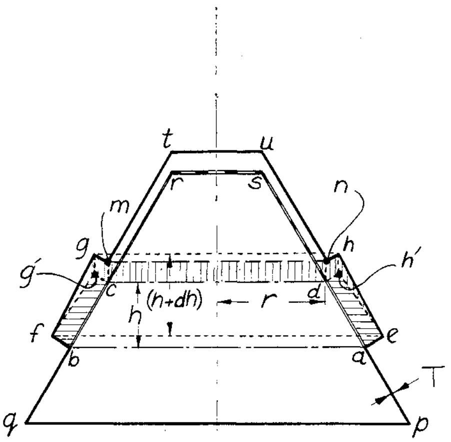

Fig. 1 Conical shell with a transverse element under strain.

The results were based upon height and volume changes, dh and dv, associated with a transverse element of the tank of infinitesimal height h. In the limit $h \rightarrow dL$. However, it has been found that the integrals of these quantities do not, in general, add up to the correct expressions for the total changes in tank height and volume, respectively. Instead, one should use alternative quantities dh_E and dv_E, which represent the height and volume increases effectively contributed to the over-all tank. Expressions for dh_E and dv_E are derived herewith. They can be substituted directly for dh and dv as used in Ref. 1. For a cylindrical tank, for which the cone angle $\phi = 0$, the expressions become identical. For small values of ϕ the error is negligible. In fact, for the numerical results presented in Fig. 6 of Ref. 1 the largest error in calculation is less than 0.05%. Thus, the conclusions of Ref. 1 are not changed.

Discussion

Consider a cone frustum, the cross-sectional view of which is represented by *pqrs* in Fig. 1. An element of infinitesmal height h, which in the limit becomes dL, is represented by *abcd*. The shell is of infinitesimal thickness T, so that membrane theory is applicable to its behavior under strain (no stress can be developed perpendicular to the tangent plane at any point on the shell, and no bending moment can be set up). Strain in the element *abcd* produces small deflections as follows:

1) Due to circumferential strain ϵ_θ, the element *abcd* expands and deflects to configuration *efg'h'*, where $g'h' = cd(1 + \epsilon_\theta)$, and $ef = ab(1 + \epsilon_\theta)$, while eh' remains equal to ad.

2) Due to meridional strain ϵ_ϕ, the element *efg'h'* stretches and deflects into the configuration *efgh* where $eh = eh'(1 + \epsilon_\phi)$, while gh remains equal to $g'h'$.

Thus, the volume of the element increases by an actual amount dv to become the volume represented by *efgh*. However, the volume of the frustum *pqrs* increases by an effective amount dv_E to become equal to that represented by *pqbfgmtunheap*. The volume represented by *cdsr* translates as a rigid body into position *mnut* without change. The effective volume increment dv_E is, therefore, equal to the volume of a solid obtained by rotating the shaded area *cbfgmnheadc* about the cone's axis of symmetry. The quantities dv_E and dv are not equal. In Ref. 1 dv and dh were used by mistake for integration between limits to obtain expressions for the total volume and height increases, respectively, of a conical frustum. The correct procedure is to integrate dv_E and dh_E instead.

Using the same nomenclature as Ref. 1—i.e., n_θ and n_ϕ are circumferential and meridional running skin loads, respectively—the expressions for dh_E and dv_E can be written as follows:

$$dh_E = (1/T \cos^2\phi)(n_\phi/E_{Mh} - n_\theta/E_{Hh})dL \tag{1}$$

$$dv_E = (\pi r^2/T \cos^2\phi)(n_\phi/E_{Mv} + n_\theta/E_{Hv})dL \tag{2}$$

where $E_{Mh} = E\phi$, $E_{Hh} = E_\theta/\nu_{\phi\theta}$, $E_{Mv} = E_\phi/(1 - 2\nu_{\theta\phi})$, and $E_{Hv} = E_\theta/(2 - \nu_{\phi\theta})$; E_ϕ and E_θ are Young's moduli in the two principal directions, and $\nu_{\phi\theta}$ and $\nu_{\theta\phi}$ are the Poisson coefficients for anisotropic material.

Equations (1) and (2) herewith are equivalent to Eqs. (3) and (4) of Ref. 1 and may be substituted directly in all subsequent parts of Ref. 1.

On page 14 of Ref. 2 Leknhitskii states that $E_\theta\nu_{\theta\phi} = E_\phi\nu_{\phi\theta}$ always. Making use of this theorem, it can be seen that when $\phi = 0$ the expressions for dh and dv as given in Eqs. (3) and (4) of Ref. 1 become identical with those of Eqs. (1) and (2) given herewith for dh_E and dv_E. Thus, for small values of ϕ the discrepancy becomes negligible.

References

[1] Pengelley, C. D., "Natural Frequency of Longitudinal Modes of Liquid Propellant Space Launch Vehicles," *Journal of Spacecraft and Rockets*, Vol. 5, No. 12, Dec. 1968, pp. 1425–1431.

[2] Lekhnitskii, S. G., *Theory of Elasticity of an Anisotropic Elastic Body*, Holden-Day, San Francisco, Calif., 1963.

Erratum: "Orbital Gyrocompassing Heading Reference"

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R. A. CARLSTROM of Huntington, New York was kind enough to point out an error in a figure I prepared. Figure 1 should replace Fig. 4 of the subject paper.

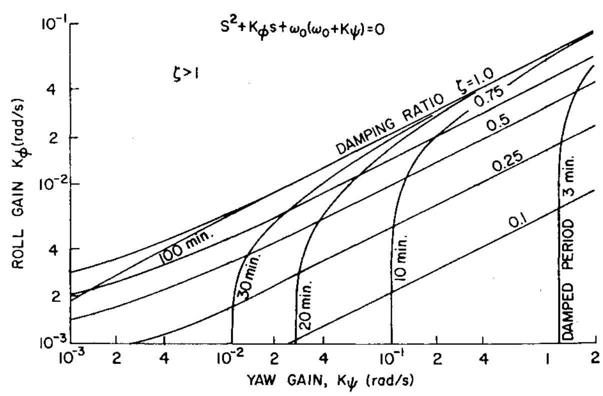

Fig. 1 Natural period and damping of gyro compass as a function of gains in Fig. 3.

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